Simplified Risk Aware CVaR-based POMDP With Performance Guarantees: a Risk Envelope Perspective Supplementary Material

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This document provides supplementary material to [1]. Therefore, it should not be considered a self-contained document, but instead regarded as an appendix of [1]. Throughout this report, all notations and definitions are with compliance to the ones presented in [1].

1 Proof Theorem 2

Denote by $\bar{\xi} \in \mathcal{U}_{cvar}(\alpha, \bar{\mathbb{P}}(z_{k+1}|b_k, a_k))$ and $\xi^* \in \mathcal{U}_{cvar}(\alpha, \mathbb{P}(z_{k+1}|b_k, a_k))$ the optimal risk ratios, considering the corresponding risk envelopes,

$$\overline{CVaR}_\alpha(\mathcal{H}(\hat{\bar{b}}_{k+1}) \mid \hat{b}_k, a_k) = \min_{\xi \in \mathcal{U}_{\text{cvar}}(\alpha, \bar{\mathbb{P}}(z_{k+1}|\hat{b}_k, a_k))} \bar{\mathbb{E}}_{z_{k+1}}[\xi(z_{k+1})\mathcal{H}(\hat{\bar{b}}_{k+1})|\hat{b}_k, a_k] =$$

$$\bar{\mathbb{E}}_{z_{k+1}}[\bar{\xi}(z_{k+1})\mathcal{H}(\hat{\bar{b}}_{k+1})|\hat{b}_k, a_k],$$

$$CVaR_\alpha(\mathcal{H}(\hat{b}_{k+1})|\hat{b}_k, a_k) = \min_{\xi \in \mathcal{U}_{\text{cvar}}(\alpha, \mathbb{P}(z_{k+1}|\hat{b}_k, a_k))} \mathbb{E}_{z_{k+1}}[\xi(z_{k+1})\mathcal{H}(\hat{b}_{k+1})|\hat{b}_k, a_k] =$$

$$\mathbb{E}_{z_{k+1}}[\xi^*(z_{k+1})\mathcal{H}(\hat{b}_{k+1})|\hat{b}_k, a_k].$$

Lower bound

We use the dual form and Lemma 2 to prove the lower bound,

$$\overline{CVaR}_\alpha(\mathcal{H}(\hat{\bar{b}}_{k+1}) \mid \hat{b}_k, a_k) - CVaR_\alpha(\mathcal{H}(\hat{b}_{k+1}) \mid \hat{b}_k, a_k) = \tag{1}$$

$$\bar{\mathbb{E}}_{z_{k+1}}(\bar{\xi}(z_{k+1})\mathcal{H}(\hat{\bar{\mathbb{P}}}(x_{k+1}|\hat{b}_k, a_k, z_{k+1}))) - \mathbb{E}_{z_{k+1}}(\xi^*(z_{k+1})\mathcal{H}(\hat{\mathbb{P}}(x_{k+1}|\hat{b}_k, a_k, z_{k+1}))) \underbrace{\geq}_{\text{Lemma 2}} \tag{2}$$

$$\bar{\mathbb{E}}_{z_{k+1}}(\bar{\xi}(z_{k+1})\mathcal{H}(\hat{\bar{\mathbb{P}}}(x_{k+1}|\hat{b}_k, a_k, z_{k+1}))) - \mathbb{E}_{z_{k+1}}(\bar{\xi}(z_{k+1})\mathcal{H}(\hat{\mathbb{P}}(x_{k+1}|\hat{b}_k, a_k, z_{k+1}))), \tag{3}$$

where we use $\hat{\mathbb{P}}(x_{k+1}|\hat{b}_k, a_k, z_{k+1})$ to explicitly denote the dependence of the particle belief $\hat{b}_{k+1}$ on $\hat{b}_k, a_k$ and z_{k+1} (and similarly for $\hat{\bar{\mathbb{P}}}(x_{k+1}|\hat{b}_k, a_k, z_{k+1})$ and $\hat{\bar{b}}_{k+1}$).

We now plug-in the entropy estimator ((34) in the main paper)

$$-\bar{\eta}_{k+1}\sum_{m=1}^{M}\bar{\xi}(z_{k+1}^m)\sum_{i=1}^{N}\bar{Z}(z_{k+1}^m|x_{k+1}^i)q_k^i\log(\frac{\bar{Z}(z_{k+1}^m|x_{k+1}^i)\sum_{j=1}^{N}T(x_{k+1}^i|x_k^i,a_k)q_k^j}{\sum_{i'=1}^{N}\bar{Z}(z_{k+1}^m|x_{k+1}^{i'})q_k^{i'}})+ \tag{4}$$

$$\bar{\eta}_{k+1}\sum_{m=1}^{M}\bar{\xi}(z_{k+1}^m)\sum_{i=1}^{N}Z(z_{k+1}^m|x_{k+1}^i)q_k^i\log(\frac{Z(z_{k+1}^m|x_{k+1}^i)\sum_{j=1}^{N}T(x_{k+1}^i|x_k^i,a_k)q_k^j}{\sum_{i'=1}^{N}Z(z_{k+1}^m|x_{k+1}^{i'})q_k^{i'}})=$$

$$-\bar{\eta}_{k+1}\sum_{c=1}^{C}\sum_{t=K\cdot(c-1)+1}^{K\cdot C}\bar{\xi}(z_{k+1}^t)\sum_{i=1}^{N}Z(z_{k+1}^t|x_{k+1}^i)q_k^i\cdot\log(\frac{\bar{Z}(z_{k+1}^t|x_{k+1}^i)\sum_{i'=1}^{N}Z(z_{k+1}^t|x_{k+1}^{i'})q_k^{i'}}{Z(z_{k+1}^t|x_{k+1}^i)\sum_{i'=1}^{N}\bar{Z}(z_{k+1}^t|x_{k+1}^{i'})q_k^{i'}}).$$

Using the inequality $\log(x)\le x-1, \forall x>0$,

$$-\bar{\eta}_{k+1}\sum_{c=1}^{C}\sum_{t=K\cdot(c-1)+1}^{K\cdot C}\bar{\xi}(z_{k+1}^t)\sum_{i=1}^{N}Z(z_{k+1}^t|x_{k+1}^i)q_k^i\cdot\log(\frac{\bar{Z}(z_{k+1}^t|x_{k+1}^i)\sum_{i'=1}^{N}Z(z_{k+1}^t|x_t^{i'})q_k^{i'}}{Z(z_{k+1}^t|x_{k+1}^i)\sum_{i'=1}^{N}\bar{Z}(z_{k+1}^t|x_{k+1}^{i'})q_k^{i'}})\ge \tag{5}$$

$$\bar{\eta}_{k+1}\sum_{c=1}^{C}\sum_{t=K\cdot(c-1)+1}^{K\cdot C}\bar{\xi}(z_{k+1}^t)\sum_{i=1}^{N}Z(z_{k+1}^t|x_{k+1}^i)q_k^i[1-\frac{\bar{Z}(z_{k+1}^t|x_{k+1}^i)\sum_{i'=1}^{N}Z(z_{k+1}^t|x_{k+1}^{i'})q_k^{i'}}{Z(z_{k+1}^t|x_{k+1}^i)\sum_{i'=1}^{N}\bar{Z}(z_{k+1}^t|x_{k+1}^{i'})q_k^{i'}}]=$$

$$\bar{\eta}_{k+1}\sum_{c=1}^{C}\sum_{t=K\cdot(c-1)+1}^{K\cdot C}\bar{\xi}(z_{k+1}^t)\sum_{i=1}^{N}Z(z_{k+1}^t|x_{k+1}^i)q_k^i-$$

$$\bar{\eta}_{k+1}\sum_{c=1}^{C}\sum_{t=K\cdot(c-1)+1}^{K\cdot C}\bar{\xi}(z_{k+1}^t)\sum_{i=1}^{N}Z(z_{k+1}^t|x_{k+1}^i)q_k^i=0.$$

Upper bound

We now prove the upper bound:

$$\overline{CVaR}_\alpha(\mathcal{H}(\hat{\bar{b}}_{k+1})\mid\hat{b}_k,a_k)-CVaR_\alpha(\mathcal{H}(\hat{b}_{k+1})\mid\hat{b}_k,a_k)= \tag{6}$$

$$\overline{\mathbb{E}}_{z_{k+1}}(\bar{\xi}(z_{k+1})\mathcal{H}(\hat{\bar{\mathbb{P}}}(x_{k+1}|\hat{b}_k,a_k,z_{k+1}))-\mathbb{E}_{z_{k+1}}(\xi^*(z_{k+1})\mathcal{H}(\hat{\mathbb{P}}(x_{k+1}|\hat{b}_k,a_k,z_{k+1})))= \tag{7}$$

$$-\bar{\eta}_{k+1}\sum_{m=1}^{M}\bar{\xi}(z_{k+1}^m)\sum_{i=1}^{N}\bar{Z}(z_{k+1}^m|x_{k+1}^i)q_k^i\log(\frac{\bar{Z}(z_{k+1}^m|x_{k+1}^i)\sum_{j=1}^{N}T(x_{k+1}^i|x_k^i,a_k)q_k^j}{\sum_{i'=1}^{N}\bar{Z}(z_{k+1}^m|x_{k+1}^{i'})q_k^{i'}})+ \tag{8}$$

$$\bar{\eta}_{k+1}\sum_{m=1}^{M}\xi^*(z_{k+1}^m)\sum_{i=1}^{N}Z(z_{k+1}^m|x_{k+1}^i)q_k^i\log(\frac{Z(z_{k+1}^m|x_{k+1}^i)\sum_{j=1}^{N}T(x_{k+1}^i|x_k^i,a_k)q_k^j}{\sum_{i'=1}^{N}Z(z_{k+1}^m|x_{k+1}^{i'})q_k^{i'}}). \tag{9}$$

We select a general element g from the risk envelope $\mathcal{U}_{\mathrm{cvar}}(\alpha,\bar{\mathbb{P}}(z_{k+1}^m|\hat{\bar{b}}_k,a_k))$. Since a risk measure involves solving a minimization problem over this envelope, considering a general element within the risk envelope makes the entire expression larger. Additionally, we define $h(m)=[\frac{m}{K}]+1$ for a general integer m. We

define g as follows and then prove it indeed belongs to $\mathcal{U}_{\text{cvar}}(\alpha, \bar{\mathbb{P}}(z_{k+1}^m | \hat{\bar{b}}_k, a_k))$:

$$g(z_{k+1}^m) \triangleq \frac{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \xi^*(z_{k+1}^t)\mathbb{P}(z_{k+1}^t | \hat{b}_k, a_k)}{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \mathbb{P}(z_{k+1}^t | \hat{b}_k, a_k)}. \tag{10}$$

We now prove that $g(z_{k+1}^m) \in \mathcal{U}_{\text{cvar}}(\alpha, \bar{\mathbb{P}}(z_{k+1}^m | \hat{\bar{b}}_k, a_k))$:

$$\sum_{m=1}^M g(z_{k+1}^m)\bar{\mathbb{P}}(z_{k+1}^m | \hat{\bar{b}}_k, a_k) = \sum_{m=1}^M \frac{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \xi^*(z_{k+1}^t)\mathbb{P}(z_{k+1}^t | \hat{b}_k, a_k)}{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \mathbb{P}(z_{k+1}^t | \hat{b}_k, a_k)} \bar{\mathbb{P}}(z_{k+1}^m | \hat{\bar{b}}_k, a_k) =$$

$$\sum_{m=1}^M \frac{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \xi^*(z_{k+1}^t)\mathbb{P}(z_{k+1}^t | \hat{b}_k, a_k)}{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \sum_{i=1}^N Z(z_{k+1}^t | x_{k+1}^i) \sum_{j=1}^N T(x_{k+1}^i | x_k^j, a_k)q_k^j} \cdot$$

$$\sum_{i=1}^N \bar{Z}(z_{k+1}^m | x_{k+1}^i) \sum_{j=1}^N T(x_{k+1}^i | x_k^j, a_k)q_k^j =$$

$$\sum_{m=1}^M \frac{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \xi^*(z_{k+1}^t)\mathbb{P}(z_{k+1}^t | \hat{b}_k, a_k)}{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \sum_{i=1}^N Z(z_{k+1}^t | x_{k+1}^i) \sum_{j=1}^N T(x_{k+1}^i | x_k^j, a_k)q_k^j} \cdot$$

$$\frac{1}{K} \sum_{i=1}^N \sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} Z(z_{k+1}^t | x_{k+1}^i) \sum_{j=1}^N T(x_{k+1}^i | x_k^j, a_k)q_k^j =$$

$$\sum_{m=1}^M \frac{1}{K} \sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \xi^*(z_{k+1}^t)\mathbb{P}(z_{k+1}^t | \hat{b}_k, a_k) = 1,$$

and

$$g(z_{k+1}^m) = \frac{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \xi^*(z_{k+1}^t)\mathbb{P}(z_{k+1}^t | \hat{b}_k, a_k)}{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \mathbb{P}(z_{k+1}^t | \hat{b}_k, a_k)} \leq \frac{1}{\alpha} \frac{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \mathbb{P}(z_{k+1}^t | \hat{b}_k, a_k)}{\sum_{t=K\cdot(h(m)-1)+1}^{K\cdot h(m)} \mathbb{P}(z_{k+1}^t | \hat{b}_k, a_k)} = \frac{1}{\alpha}. \tag{11}$$

To conclude, we showed that $\sum_{m=1}^M g(z_{k+1}^m)\bar{\mathbb{P}}(z_{k+1}^m | \hat{\bar{b}}_k, a_k) = 1$ and $g(z_{k+1}^m) \leq \frac{1}{\alpha}$. Therefore, according to (11), $g(z_{k+1}^m) \in \mathcal{U}_{\text{cvar}}(\alpha, \bar{\mathbb{P}}(z_{k+1}^m | \hat{\bar{b}}_k, a_k))$.

Referring to (6), we now take notice that $\log(\frac{Z(z_{k+1}^m | x_{k+1}^i) \sum_{j=1}^N T(x_{k+1}^i | x_k^i, a_k)q_k^j}{\sum_{i'=1}^N Z(z_{k+1}^m | x_{k+1}^{i'})q_k^{i'}}) < 0$, as the log of a discrete probability distribution. By replacing the optimal risk ratio $\bar{\xi}(z_{k+1}^m) \in \mathcal{U}_{\text{cvar}}(\alpha, \bar{\mathbb{P}}(z_{k+1}^m | \hat{\bar{b}}_k, a_k))$ with the risk ratio $g(z_{k+1}^m) \in \mathcal{U}_{\text{cvar}}(\alpha, \bar{\mathbb{P}}(z_{k+1}^m | \hat{\bar{b}}_k, a_k))$, in (6), we get the bound

$$\overline{CVaR}_\alpha(\mathcal{H}(\hat{\bar{b}}_{k+1}) \mid \hat{b}_k, a_k) - CVaR_\alpha(\mathcal{H}(\hat{b}_{k+1}) \mid \hat{b}_k, a_k) \leq \tag{12}$$

$$-\bar{\eta}_{k+1} \sum_{m=1}^M g(z_{k+1}^m) \sum_{i=1}^N \bar{Z}(z_{k+1}^m | x_{k+1}^i)q_k^i \log(\frac{\bar{Z}(z_{k+1}^m | x_{k+1}^i) \sum_{j=1}^N T(x_{k+1}^i | x_k^i, a_k)q_k^j}{\sum_{i'=1}^N \bar{Z}(z_{k+1}^m | x_{k+1}^{i'})q_k^{i'}}) + \tag{13}$$

$$\bar{\eta}_{k+1} \sum_{m=1}^M \xi^*(z_{k+1}^m) \sum_{i=1}^N Z(z_{k+1}^m | x_{k+1}^i)q_k^i \log(\frac{Z(z_{k+1}^m | x_{k+1}^i) \sum_{j=1}^N T(x_{k+1}^i | x_k^i, a_k)q_k^j}{\sum_{i'=1}^N Z(z_{k+1}^m | x_{k+1}^{i'})q_k^{i'}}). \tag{14}$$

We now look at one of the clusters (without loss of generality, at the first one, i.e. $m = 1$), plug-in the definition of the abstraction observation model (29), and define a matrix notation of the cluster,

$$g(z_{k+1}^{m=1}) \sum_{i=1}^{N} \bar{Z}(z_{k+1}^{t}|x_{k+1}^{i})q_{k}^{i} = \frac{\sum_{t=1}^{K} \xi^{*}(z_{k+1}^{t})\mathbb{P}(z_{k+1}^{t}|\hat{b}_{k},a_{k})}{\sum_{t=1}^{K} \mathbb{P}(z_{k+1}^{t}|\hat{b}_{k},a_{k})} \sum_{i=1}^{N} \bar{Z}(z_{k+1}^{t}|x_{k+1}^{i})q_{k}^{i} \triangleq \frac{1}{K} \frac{\xi^{T}P\mathbf{1}^{T}Z_{q}}{\mathbf{1}^{T}P},$$
(15)

where

$$\xi \triangleq \begin{bmatrix} \xi^{*}(z_{k+1}^{1}) \\ \cdot \\ \cdot \\ \xi^{*}(z_{k+1}^{K}) \end{bmatrix}, Z_{q} \triangleq \begin{bmatrix} \sum_{i=1}^{N} Z(z_{k+1}^{1}|x_{k+1}^{i})q_{k}^{i} \\ \cdot \\ \cdot \\ \sum_{i=1}^{N} Z(z_{k+1}^{K}|x_{k+1}^{i})q_{k}^{i} \end{bmatrix}, P \triangleq \begin{bmatrix} \mathbb{P}(z_{k+1}^{1}|\hat{b}_{k},a_{k}) \\ \cdot \\ \cdot \\ \mathbb{P}(z_{k+1}^{K}|\hat{b}_{k},a_{k}) \end{bmatrix}, \mathbf{1} \triangleq \begin{bmatrix} 1 \\ \cdot \\ \cdot \\ 1 \end{bmatrix}.$$
(16)

We apply

$$\frac{1}{K}\frac{\xi^{T}P\mathbf{1}^{T}Z_{q}}{\mathbf{1}^{T}P} = \frac{1}{K}\frac{tr(\xi^{T}P\mathbf{1}^{T}Z_{q})}{tr(\mathbf{1}^{T}P)} \leq \frac{1}{K}\frac{tr(P\mathbf{1}^{T})tr(Z_{q}\xi^{T})}{tr(\mathbf{1}^{T}P)} = \frac{1}{K}tr(Z_{q}\xi^{T}) = \xi^{T}Z_{q}.$$
(17)

Hence, we re-write (13) as

$$-\bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^{*}(z_{k+1}^{m}) \sum_{i=1}^{N} Z(z_{k+1}^{m}|x_{k+1}^{i})q_{k}^{i} \log(\frac{\bar{Z}(z_{k+1}^{m}|x_{k+1}^{i}) \sum_{j=1}^{N} T(x_{k+1}^{i}|x_{k}^{i},a_{k})q_{k}^{j}}{\sum_{i'=1}^{N} \bar{Z}(z_{k+1}^{m}|x_{k+1}^{i'})q_{k}^{i'}})+$$
(18)

$$\bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^{*}(z_{k+1}^{m}) \sum_{i=1}^{N} Z(z_{k+1}^{m}|x_{k+1}^{i})q_{k}^{i} \log(\frac{Z(z_{k+1}^{m}|x_{k+1}^{i}) \sum_{j=1}^{N} T(x_{k+1}^{i}|x_{k}^{i},a_{k})q_{k}^{j}}{\sum_{i'=1}^{N} Z(z_{k+1}^{m}|x_{k+1}^{i'})q_{k}^{i'}}) =$$

$$\underbrace{\bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^{*}(z_{k+1}^{m}) \sum_{i=1}^{N} Z(z_{k+1}^{m}|x_{k+1}^{i})q_{k}^{i} \log(\frac{Z(z_{k+1}^{m}|x_{k+1}^{i})}{\bar{Z}(z_{k+1}^{m}|x_{k+1}^{i})}) +}_{(a)}$$

$$\underbrace{\bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^{*}(z_{k+1}^{m}) \sum_{i=1}^{N} Z(z_{k+1}^{m}|x_{k+1}^{i})q_{k}^{i} \log(\frac{\sum_{i=1}^{N} Z(z_{k+1}^{m}|x_{k+1}^{i})q_{k}^{i}}{\sum_{i=1}^{N} \bar{Z}(z_{k+1}^{m}|x_{k+1}^{i})q_{k}^{i}})}_{(b)}.$$

We now treat the terms (a) and (b) separately, starting with (a):

$$(a) = \bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^{*}(z_{k+1}^{m}) \sum_{i=1}^{N} Z(z_{k+1}^{m}|x_{k+1}^{i})q_{k}^{i} \log(K)+$$

$$\bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^{*}(z_{k+1}^{m}) \sum_{i=1}^{N} Z(z_{k+1}^{m}|x_{k+1}^{i})q_{k}^{i} \log(\frac{Z(z_{k+1}^{m}|x_{k+1}^{i})}{\sum Z(z_{k+1}^{m}|x_{k+1}^{i})})$$

$$\leq \bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^{*}(z_{k+1}^{m}) \sum_{i=1}^{N} Z(z_{k+1}^{m}|x_{k+1}^{i})q_{k}^{i} \log(K) = \log(K).$$

For term (b), we utilize the Jensen's inequality,

$$
\begin{aligned}
(b) &= \bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^*(z_{k+1}^m) \sum_{i=1}^{N} Z(z_{k+1}^m | x_{k+1}^i) q_k^i \log\Big(\frac{\sum_{i=1}^{N} \bar{Z}(z_{k+1}^m | x_{k+1}^i) q_k^i}{\sum_{i=1}^{N} Z(z_{k+1}^m | x_{k+1}^i) q_k^i} \Big) \\
&\leq \log\Big(\bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^*(z_{k+1}^m) \sum_{i=1}^{N} Z(z_{k+1}^m | x_{k+1}^i) q_k^i \big(\frac{\sum_{i=1}^{N} \bar{Z}(z_{k+1}^m | x_{k+1}^i) q_k^i}{\sum_{i=1}^{N} Z(z_{k+1}^m | x_{k+1}^i) q_k^i} \big) \Big) \\
&= \log\Big(\bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^*(z_{k+1}^m) \sum_{i=1}^{N} \bar{Z}(z_{k+1}^m | x_{k+1}^i) q_k^i \Big).
\end{aligned}
$$

We again look at one of the clusters in matrix notations (without loss of generality, at the first),

$$
\frac{1}{K} tr((\mathbf{1}^T \xi)(\mathbf{z}^T \mathbf{1})) \leq \frac{1}{K} tr(\xi \mathbf{z}^T) tr(\mathbf{1}\mathbf{1}^T) = tr(\xi \mathbf{z}^T). \tag{19}
$$

Hence, for the term (b) we get,

$$
\log\Big(\bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^*(z_{k+1}^m) \sum_{i=1}^{N} \bar{Z}(z_{k+1}^m | x_{k+1}^i) q_k^i \Big) \leq \log\Big(\bar{\eta}_{k+1} \sum_{m=1}^{M} \xi^*(z_{k+1}^m) \sum_{i=1}^{N} Z(z_{k+1}^m | x_{k+1}^i) q_k^i \Big) = 0.
$$

$$\tag{20}$$

Therefore, $0 \leq \overline{CVaR}_\alpha(\mathcal{H}(\hat{\bar{b}}_{k+1}) \mid \hat{b}_k, a_k) - CVaR_\alpha(\mathcal{H}(\hat{b}_{k+1}) \mid \hat{b}_k, a_k) \leq \log(K)$.

References

1. I. Nutov and V. Indelman. Simplified risk aware cvar-based pomdp with performance guarantees: a risk envelope perspective. In *Proc. of the Intl. Symp. of Robotics Research (ISRR)*, 2024. Submitted.