

# Simplified Online Planning Under Uncertainty with Performance Guarantees

Vadim Indelman











In Cooperation with









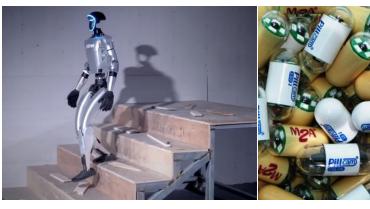


# **Advanced Autonomy**

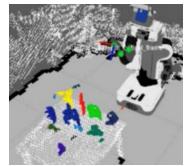
Involves autonomous navigation, active SLAM, informative gathering, active sensing, etc.



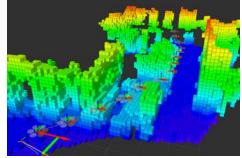


















# **Advanced Autonomy**

# Perception and Inference

Where am I? What is the surrounding environment?



# **Decision-Making Under Uncertainty**

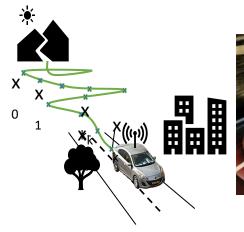
What should I be doing next?

Determine best action(s) to accomplish a task, account for different sources of uncertainty

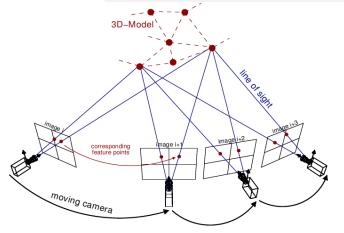
### **Perception and Inference**

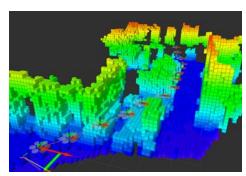


### **Decision-Making Under Uncertainty**









# Challenge

#### **Probabilistic Inference**

Maintain a distribution over the state given data

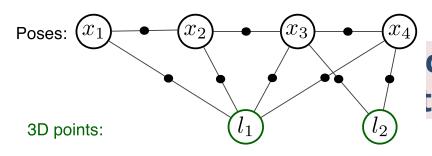
$$b_k \triangleq b[X_k] = \mathbb{P}(X_k \mid a_{0:k-1}, z_{1:k})$$
state actions observations

### **Decision-making under uncertainty**

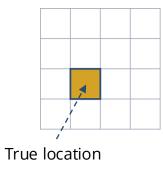
Involves reasoning about the entire observation and action spaces along planning horizon

### **Computationally intractable**

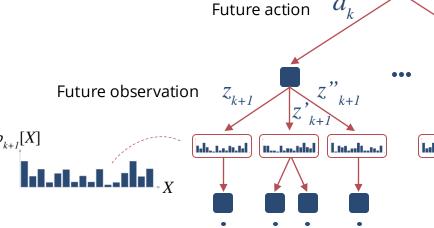
More so, in high dimensional settings

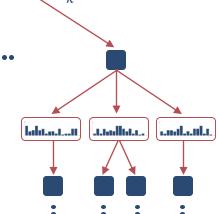


#### Example - grid world





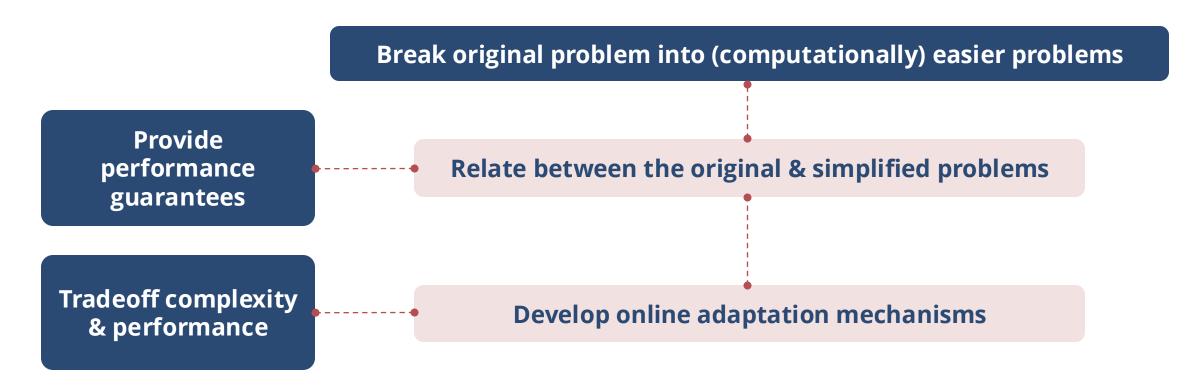




ct autonomously online and efficiently tasks in a safe and reliable fashion??

# Simplification Framework

Accelerate decision making by adaptive simplification while providing performance guarantees



$$\mathcal{LB}(b,a) \leq Q(b,a) \leq \mathcal{UB}(b,a)$$
 Computationally cheap(er) bounds

### **Concept**:

- Identify and solve a simplified (computationally) easier decision-making problem
- Provide performance guarantees

### **Specific simplifications include**:

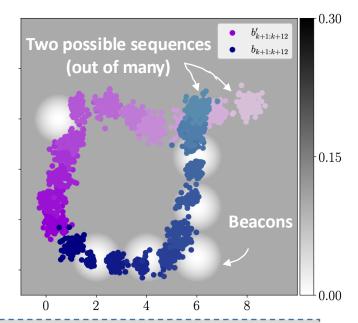
- Sparsification of Gaussian beliefs (high dim. state)
- Topological metric for Gaussian beliefs (high dim. state)
- Utilize a subset of samples (nonparametric beliefs)
- Utilize a subset of hypotheses (hybrid beliefs)

- Simplified models and spaces
- Simplification of Risk-Averse POMDP Planning
- Simplification in a multi-agent setting

# Simplification of POMDPs with Nonparametric Beliefs

Value function

$$V^{\pi}(b_0) \triangleq \mathbb{E}\left[\sum_t \gamma^t r_t(b_t, a_t) \mid a_t = \pi_t(b_t)\right]$$



### **Simplification:**

- Utilize a subset of samples for planning
- Information-theoretic reward (entropy)
- Analytical (cheaper) bounds over the reward

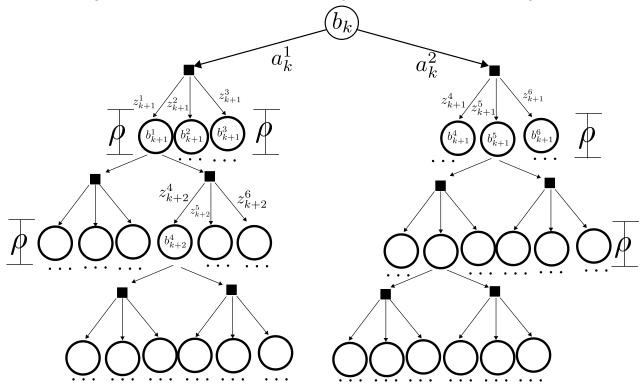
$$b = \left\{x^{i}, w^{i}\right\}_{i=1}^{N}$$

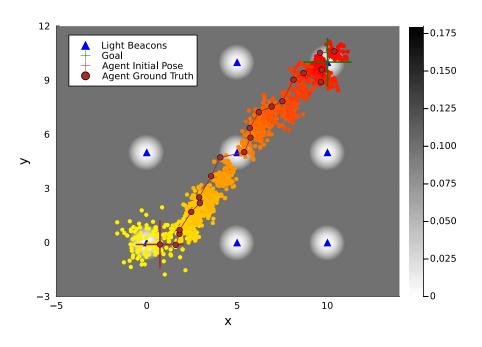
$$b^{s} = \left\{x^{j}, w^{j}\right\}_{j=1}^{N^{s}}$$
Simplifictation

$$lb(b, b^s, a) \le r(b, a) \le ub(b, b^s, a)$$

# Simplification of POMDPs with Nonparametric Beliefs

Adaptive multi-level simplification in a Sparse Sampling setting:

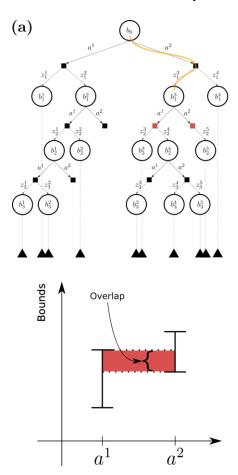




Typical speedup of 20% - 50%, Same performance!

# Simplification of POMDPs with Nonparametric Beliefs

Adaptive multi-level simplification in an MCTS setting:



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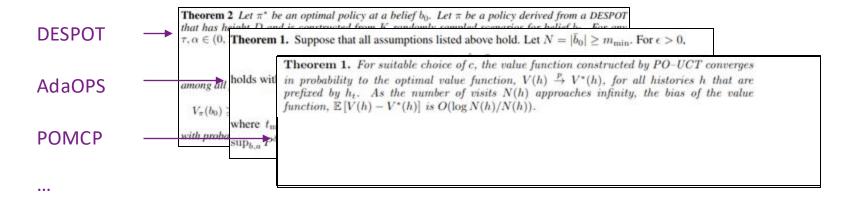
### **Specific simplifications include**:

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### POMDPs with Deterministic Guarantees

SOTA sampling based approaches come with probabilistic theoretical guarantees



Can we get deterministic guarantees?

We show that deterministic guarantees are indeed possible!

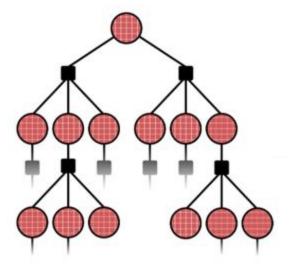
### Online POMDP Planning with Anytime Deterministic Guarantees

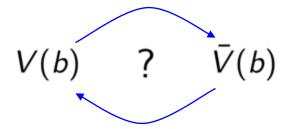
### Concept:

Instead of solving the original POMDP, consider a simplified version of that POMDP.



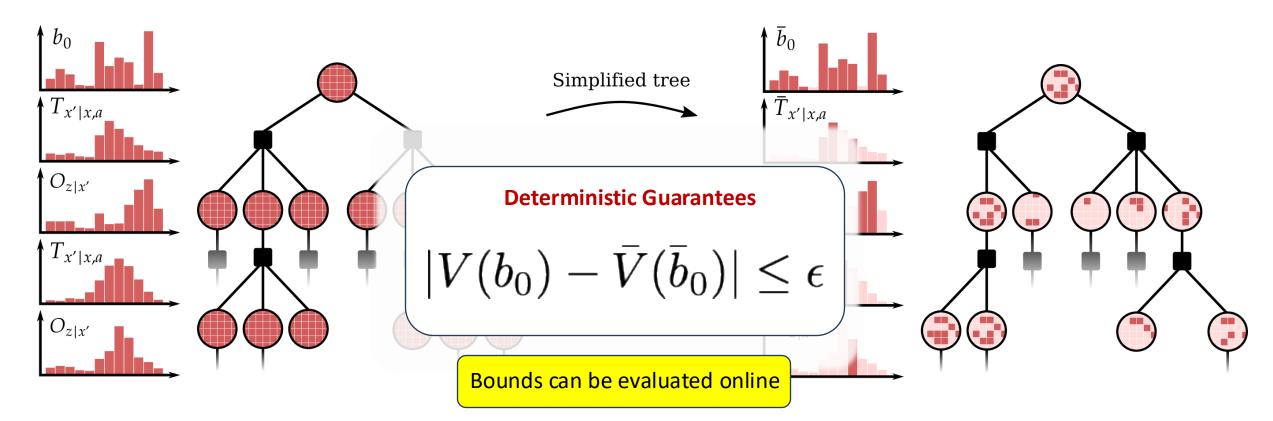
Derive a mathematical relationship between the solution of the simplified, and the theoretical POMDP.





# Online POMDP Planning with Anytime Deterministic Guarantees

Deterministic guarantees (assuming discrete spaces)



# Online POMDP Planning with Anytime Deterministic Guarantees

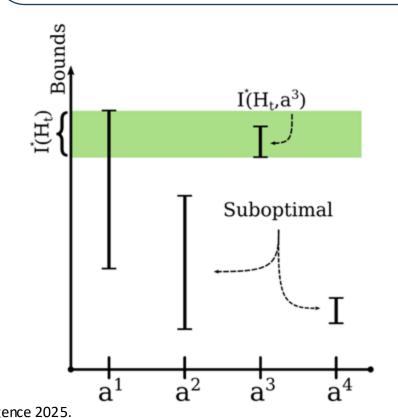
Importantly, the bounds can be calculated during planning.

How can we use them?

- Pruning of sub-optimal branches
  - Made possible by the deterministic guarantees
- Stopping criteria for the planning phase
  - Made possible by the deterministic guarantees
- Finding the optimal solution in finite time
  - Without recovering the theoretical tree

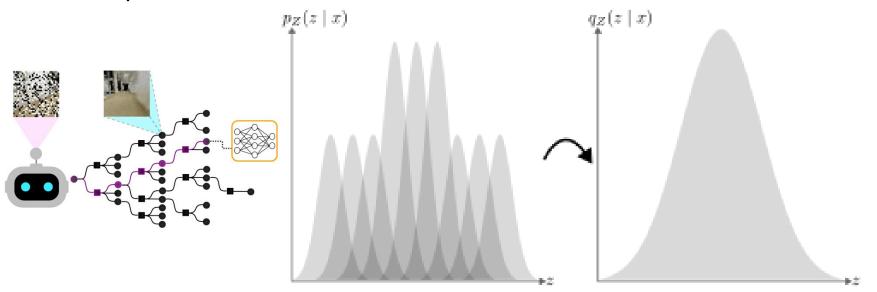
#### **Deterministic Guarantees**

$$|V(b_0) - \bar{V}(\bar{b}_0)| \le \epsilon$$



### Simplifying Complex Observation Models with Probabilistic Guarantees

- We replace the (learned) observation model  $p_Z$  with a cheaper model  $q_Z$ 
  - Simpler GMM, Shallower Neural Network, etc.
  - Example:



Simplified models  $p_{\theta}(z \mid x)$  Original, expensive  $q_{\phi}(z \mid x)$  Simplified, cheap

Can we simplify the learned models?
What is the impact on planning performance?

### Simplifying Complex Observation Models with Probabilistic Guarantees

- We replace the (learned) observation model  $p_Z$  with a cheaper model  $q_Z$
- Simplified action-value function:  $Q_{\mathbf{P}}^{q_Z}$

### Corollary 3

For arbitrary  $\varepsilon, \delta > 0$  there exists a number of particles for which

$$|Q_{\mathbf{P}}^{p_Z}(b_t, a) - \hat{Q}_{\mathbf{M}_{\mathbf{P}}}^{q_Z}(\bar{b}_t, a)| \le \hat{\Phi}_{\mathbf{M}_{\mathbf{P}}}(\bar{b}_t, a) + \varepsilon$$

 $|Q_{\mathbf{P}}^{p_Z}(b_t,a) - \hat{Q}_{\mathbf{M_P}}^{q_Z}(\bar{b}_t,a)| \leq \hat{\Phi}_{\mathbf{M_P}}(\bar{b}_t,a) + \varepsilon$  with probability of at least  $1 - \delta$  for any guaranteed planner

**Theoretical** Q function of the POMDP, with original models

**Estimator** of the Q function of a particle-belief POMDP, with **simplified** models

# Robust Online Planning Under Uncertainty

- So far, models were assumed to be given and perfect
- In practice, models are learned from data
- What happens when the models are uncertain?

How to do online robust planning?

#### **Uncertainty set:**

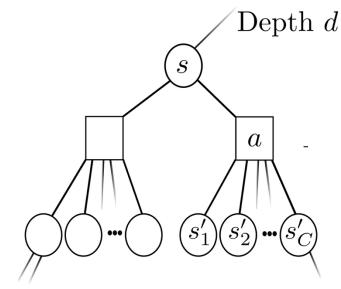
$$P_t(S_{t+1} \mid S_t = s, A_t = a) \in \mathcal{P}_t^{s,a}$$

#### **Robust value function:**

$$V^{\pi}(s) = \min_{P \in \mathcal{P}} V^{\pi, P}(s)$$

### **Robust Sparse Sampling (RSS) Algorithm:**

- A sample-based online robust planner
- Applicable to infinite or continuous state spaces
- Finite-sample performance guarantees



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Depth d

### **Robust Sparse Sampling (RSS) Algorithm:**

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#### **Prob. Guarantees**

$$\left| V^{\hat{\pi}^{\star}}(s) - V^{\pi^{\star}}(s) \right| \le \epsilon$$

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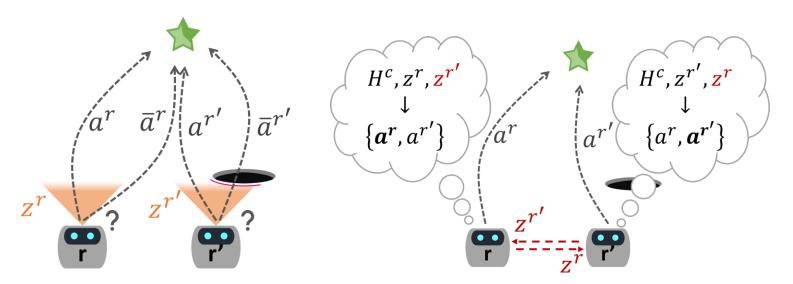
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# Multi-Robot Belief Space Planning

- A common assumption: Beliefs of different robots are consistent at planning time
- Requires prohibitively frequent data-sharing capabilities!







# Multi-Robot Cooperative BSP with Inconsistent Beliefs

### What happens when data-sharing capabilities between the robots are limited?

• Histories & beliefs of the robots may <u>differ</u> due to limited data-sharing capabilities

$$b_k^r = \mathbb{P}(x_k \mid \mathcal{H}_k^r) \qquad \qquad b_k^{r'} = \mathbb{P}(x_k \mid \mathcal{H}_k^{r'}) \qquad \qquad \mathcal{H}_k^r \neq \mathcal{H}_k^{r'}$$
 Available only to robot r Common history, e.g. from the last data-sharing

T. Kundu, M. Rafaeli, and V. Indelman, "Multi-Robot Communication-Aware Cooperative Belief Space Planning with Inconsistent Beliefs: An Action-Consistent Approach," IROS'24.

T. Kundu, M. Rafaeli, A. Gulyaev, and V. Indelman, "Action-Consistent Decentralized Belief Space Planning with Inconsistent Beliefs and Limited Data Sharing: Framework and Simplification Algorithms with Formal Guarantees," Submitted 2025.

M. Rafaeli, and V. Indelman, "Towards Optimal Performance and Action Consistency Guarantees in Dec-POMDPs with Inconsistent Beliefs and Limited Communication," Submitted, 2025.

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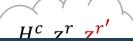
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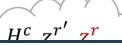
$$\mathcal{H}_k^r \neq \mathcal{H}_k^{r'}$$

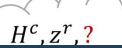
Can lead to a lack of coordination and unsafe and sub-optimal actions













 $H^c.z^{r'}$ ,?

### Challenge:

- **Guarantee** a consistent joint action selection by individual robots **despite** inconsistent histories
- Otherwise, self-trigger communication





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See additional research directions on our website!

Feel free to reach out to explore research opportunities!